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Earthquake response spectral analysis of an earth dam in the eastern United States

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ABSTRACT: During the New Hampshire earthquake of 18 January 1982 (M = 4.7) which occurred at latitude 43.5° N, longitude 71.6° W, three records were obtained on crest, abutment, and downstream sites at the Franklin Falls Dam about 8 km from the epicenter. The transverse component of the accelerograph located on the right abutment recorded a peak acceleration of 0.55 g, which is the highest acceleration ever recorded in the eastern United States. The purposes of this paper were to study the seismic response of the dam as shown by response spectra on crest, abutment, and downstream sites, and to determine the natural periods of foundation and dam from the amplitude ratios (amplification factor) of the response spectra.

1 INTRODUCTION

At 19:14:42 EST on 18 January 1982 or 00:13:42 UT on 19 January 1982, an earthquake of Richter magnitude 4.7 occurred at latitude 43.5° N, longitude 71.6° W in New Hampshire. The focal depth was estimated to be between 4.5 and 8.0 km by the Weston Observatory and US Geological Survey, respectively. A typical comment by people of Laconia, Franklin, and Tilton Northfield areas was, "The whole building shook and rumbled." The earthquake was felt in most of New England and parts of New York for about 20 seconds during the night. The Franklin Falls Dam (Fig. 1) is at a distance of 8 km from the epicenter.

Thirty-six accelerograms were recorded at five Corps of Engineers (CE) dams: Franklin Falls Dam (FFD, epicentral distance, 8 km), Union Village Dam (UVD, e.d., 60 km), North Hartland Dam (NHD, e.d., 61 km), North Springfield Dam (NSD, e.d., 76 km), Ball Mountain Dam (BMD, e.d., 103 km); and at the White River Junction (WRJ, e.d., 60 km), Veterans Administration (VA) Hospital. However, the accelerographs at Townshend Dam, Surry Dam, and at the Manchester VA Hospital were not triggered. The locations of these sites and the epicenter are shown in Fig. 2.

2 PURPOSE

An earlier report by Chang (1983) discusses the corrections of baseline and instrument for the 36 accelerograms, the integration of particle velocity and displacement, the analysis of maximum acceleration, velocity, displacement, and spectrum intensity, and the study of the attenuation rate of these four parameters with distance for various site conditions. The purpose of this paper is to analyze the observed spectral response at crest, right abutment, and free field downstream of the Franklin Falls Dam.

3 DESCRIPTION AND GEOLOGICAL ENVIRONMENT OF THE FRANKLIN FALLS DAM

The Franklin Falls Dam, located about 4 km north of Franklin, New Hampshire, is one of the flood control reservoirs for the Merrimack River Basin. The dam is of rolled earth fill with dumped rock shell and toe, 1,740 feet (530 m) long, with a maximum height of 136 feet (41.45 m) and containing about 3,300,000 cubic yards (2,500,000 m^3) of earth and rock fill. A 550-foot (168 m) spillway and 732 square feet (68 m^2) of gate-controlled outlet conduits involving 95,000 cubic yards (72,600 cubic meters) of concrete were constructed in open cut rock.

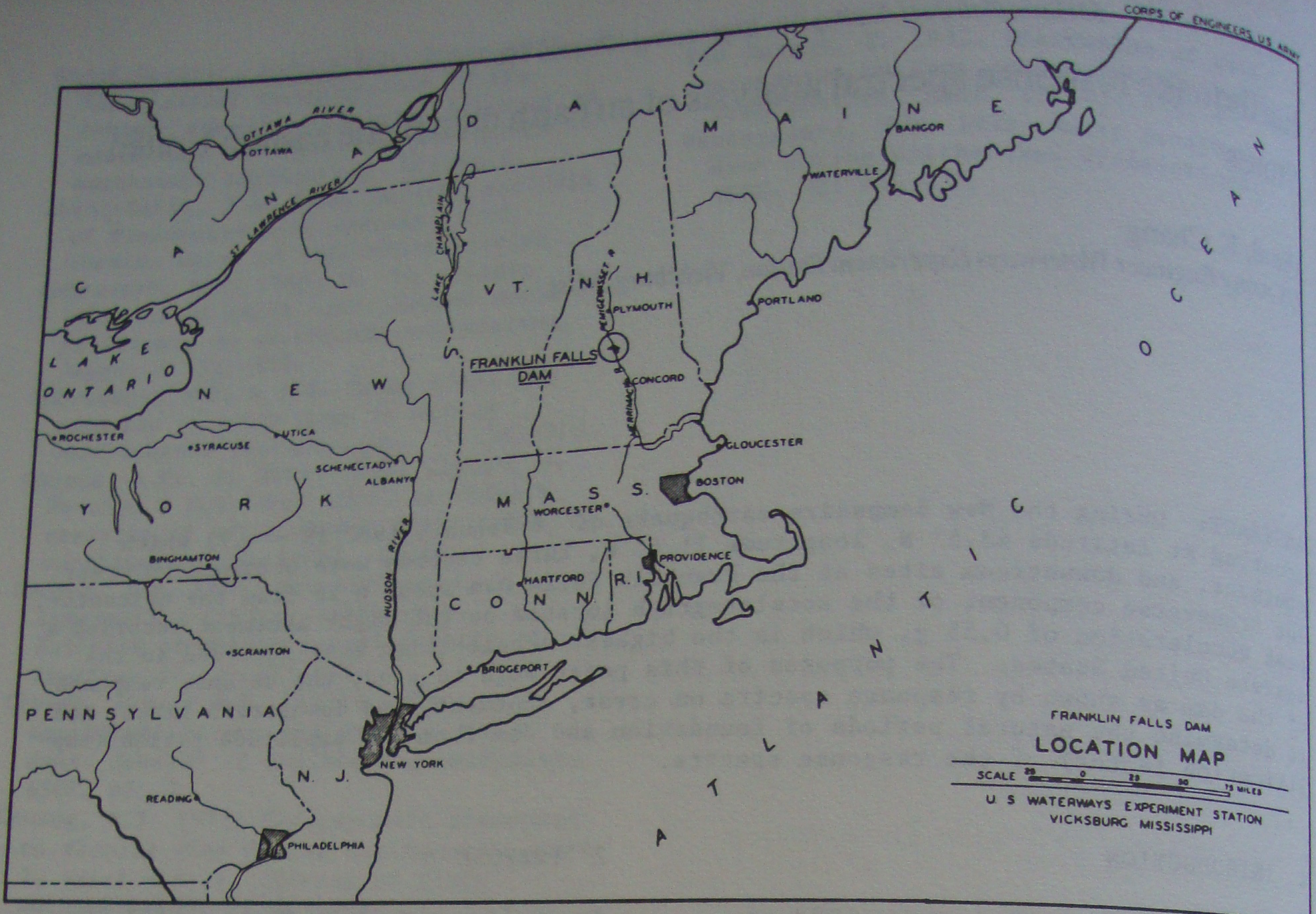

Figure 1. Location of the Franklin Falls Dam

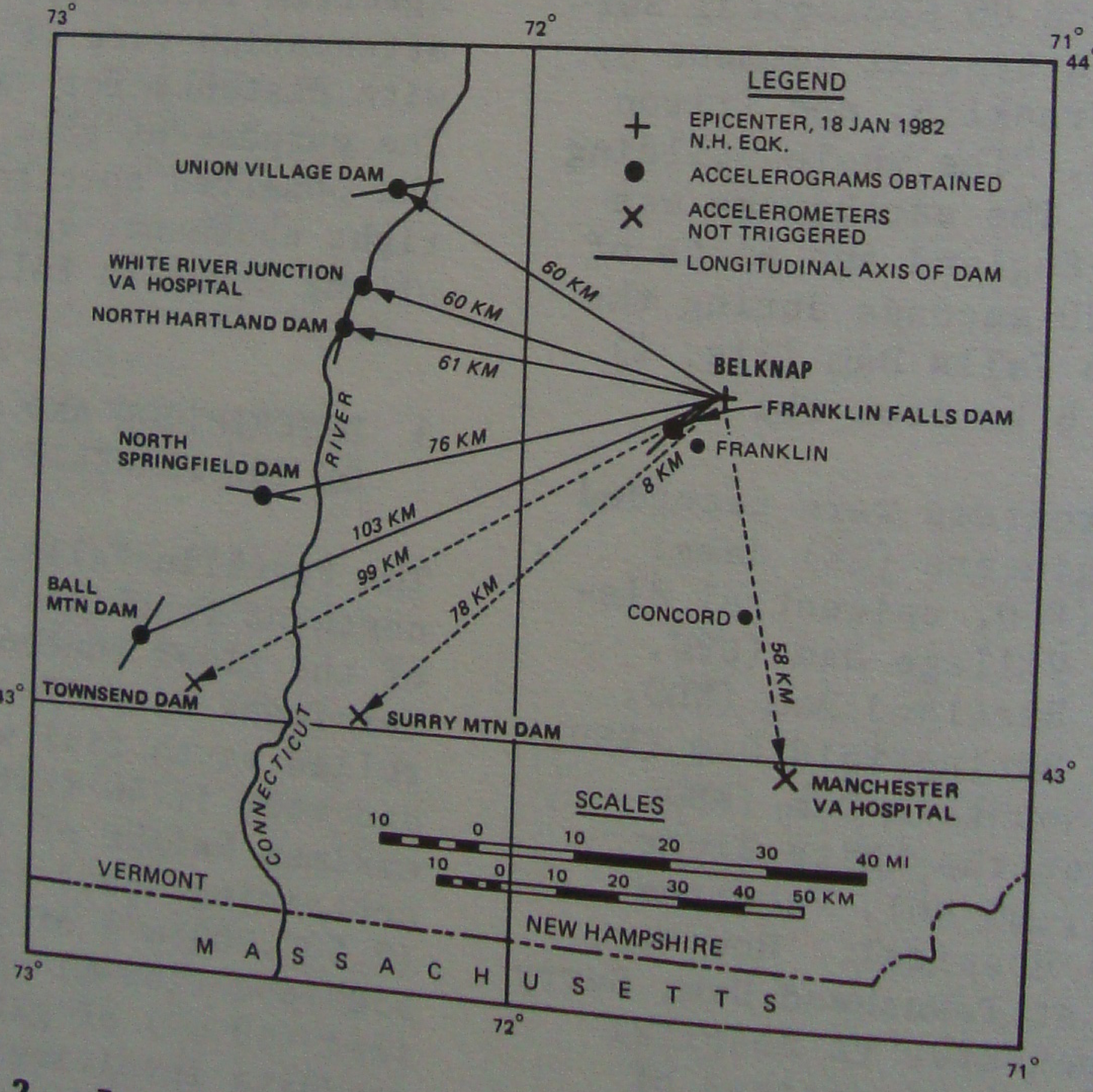

Figure 2. Locations of epicenter and recording stations

Table 1. The uncorrected and corrected strong-motion parameters of Franklin Falls Dam

Site location	Azimuth of component	Uncorrected accelerations cm/sec	Corrected: Maximum accelerations (a), cm/sec	Corrected: Maximum veloc-ity (v) cm/sec	Corrected: Maximum displace-ment (d) cm	Corrected: Velocity spectrum intensity 5%, ζ
Downstream	L-225°	111.68 (21 Hz)	140.70 (0.143 g)	2.03	0.16	9.53
	Up	208.34 (21 Hz)	271.00 (0.276 g)	1.73	0.08	7.29
	T-135°	267.94 (16 Hz)	377.86 (0.385 g)	2.87	0.17	13.23
Right Abutment	L-45°	282.52 (14 Hz)	287.70 (0.293 g)	2.68	0.25	13.08
	Up	171.89 (20 Hz)	172.89 (0.176 g)	1.86	0.41	11.61
	T-315°	565.05 (14 Hz)	539.96 (0.550 g)	5.59	0.43	22.39
Crest	L-45°	102.79 (11.4 Hz)	123.96 (0.126 g)	2.67	0.36	12.09
	Up	111.38 (11.4 Hz)	114.31 (0.116 g)	2.89	0.47	13.15
	T-315°	243.46 (14 Hz)	306.83 (0.312 g)	4.06	0.33	17.12

At the Franklin Falls Dam site, the Pemigewasset Valley has a flat bottom U-shaped cross section lying entirely on glacial deposits. The right abutment of the bedrock is a solid, unweathered rock varying between micaceous gneiss and granular mica schist, with numerous but minor veins of coarse granite. The left abutment is a steep overburden slope rising more than 150 feet (45 m) at an angle of 34 degrees.

4 THE ACCELEROGRAMS

Nine accelerograms were recorded on three accelerographs at downstream free field, right abutment, and crest sites of the Franklin Falls Dam. The uncorrected and corrected strong-motion parameters of Franklin Falls Dam are listed in Table 1. After the instrument and baseline corrections, the nine corrected accelerograms were integrated to obtain velocity and displacement records (Fig. 3 - Fig. 5). Next, the absolute acceleration response spectrum, relative velocity response spectrum, and relative displacement response spectrum for each component were calculated and plotted (Fig. 6 - 14).

5 CHARACTERISTICS OF RESPONSE SPECTRA

5.1 Absolute acceleration response spectrum

Figures 6a to 14a show that the highest peak spectral acceleration is always in the frequency range (≥ 10 Hz) of compression (P) waves; the secondary of peak amplitudes is in shear (S) waves (2 Hz < f < 10 Hz). The surface (Rayleigh, R or Love, L) waves (f < 2 Hz) appear as very small peak amplitudes, often not easily recognized. The spectral amplitude approaches zero as the period increases.

5.2 Relative velocity response spectrum

As in the case of absolute acceleration response, the compression waves usually show the highest spectral amplitudes, though the shear waves contain more energy (Fig. 6b to 14b). However, when the resonant period of the S-wave appears, the amplitudes of compression and other waves will be comparatively reduced, as demonstrated in the longitudinal component (L) of the downstream station (Fig. 12b to 14b), where the shear wave resonant period of 0.4 second (2.5 Hz) becomes the largest amplitude on the relative velocity response spectrum. The 0.4-second period is the natural period of the alluvial deposit at the toe or foundation of the Franklin Falls Dam. This resonant period of 0.4 second appears again on the L-component of the crest recording station (Fig. 9b). It is noted that the amplification factor of the L-component of the accelerograph on the crest caused by the height of the dam is less than or equal to one (Table 2). The amplitude of the relative velocity response approaches the base velocity (peak ground velocity) at period between 1 and 4 seconds (Fig. 6b - 14b).

5.3 Relative displacement response spectrum

The surface waves or the long period waves (R- and L-waves) are the dominant waves in

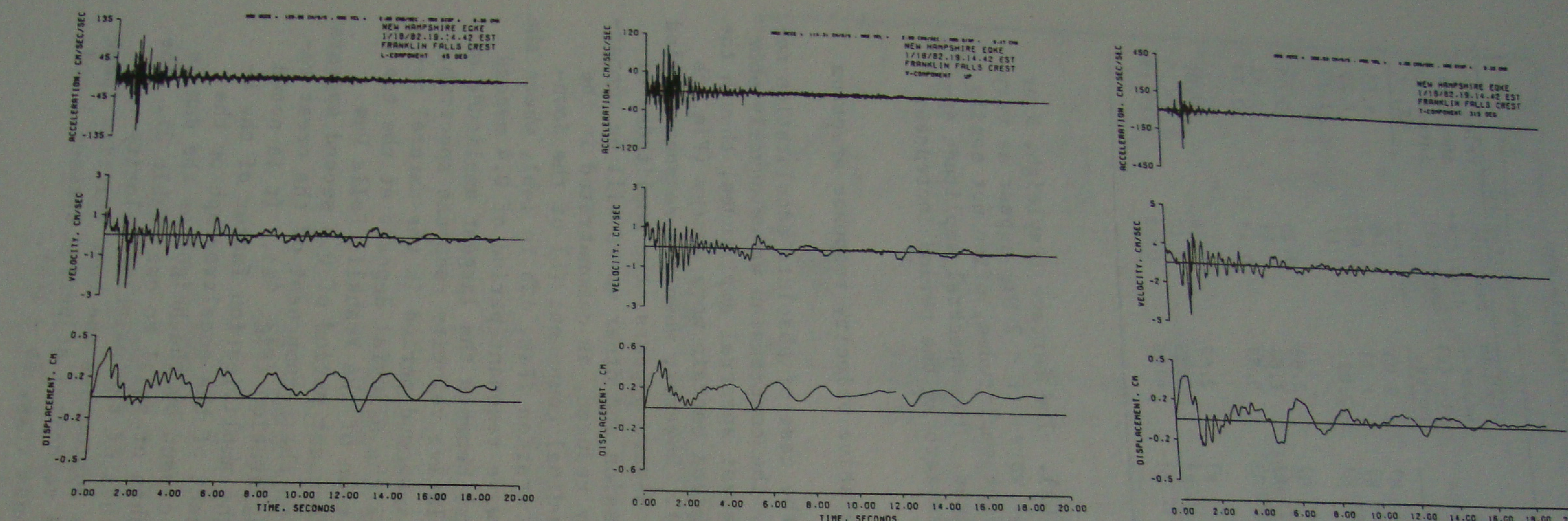

Figure 3. Corrected acceleration, velocity, and displacement of L, V, and T Components on crest

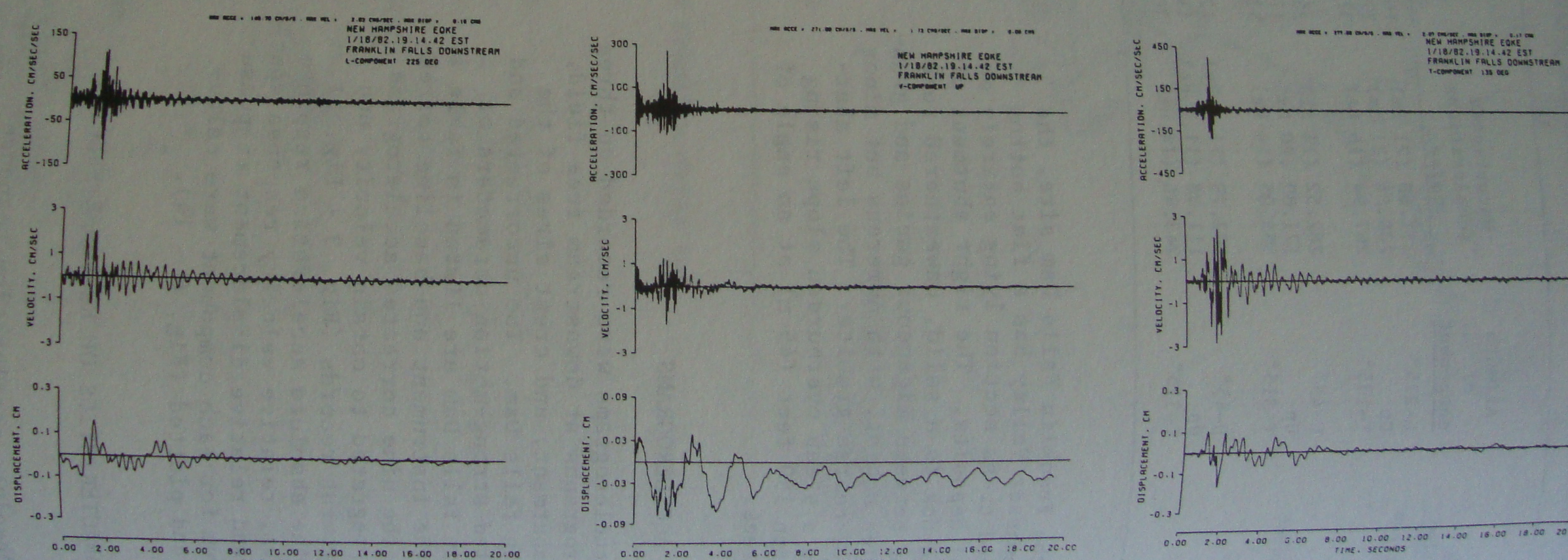

Figure 4. Corrected acceleration, velocity, and displacement of *L, V, and T Components on Downstream Free Field*

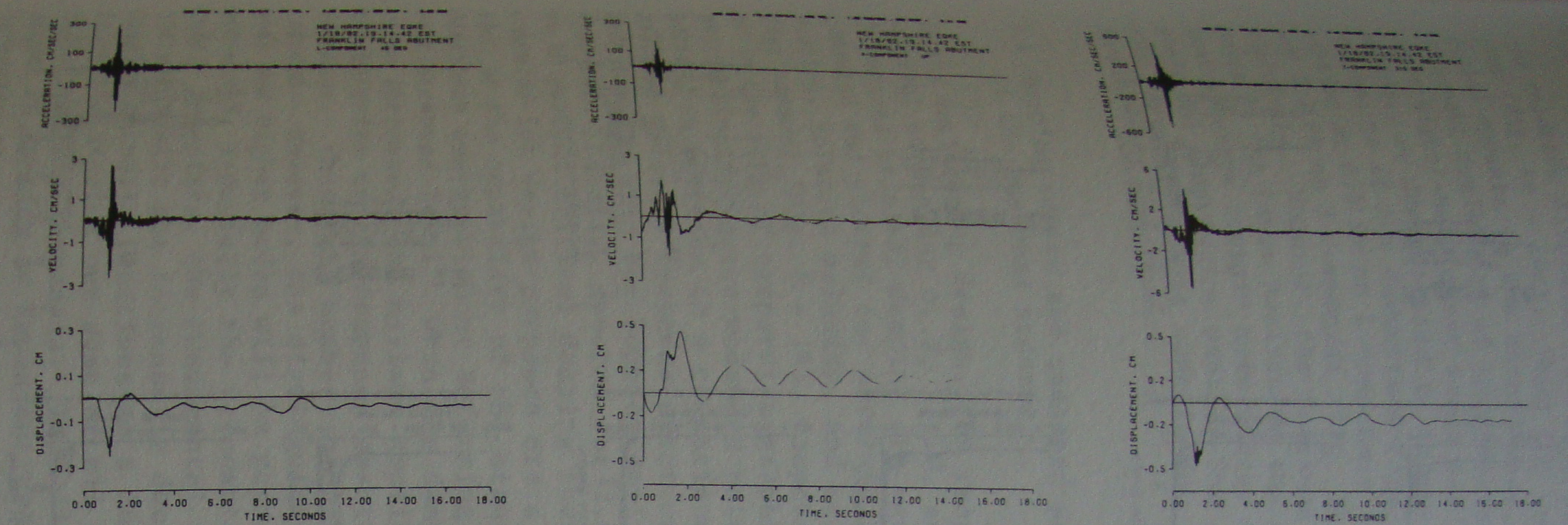

Figure 5. Corrected acceleration, velocity, and displacement of L, V, and T Component on Right Abutment

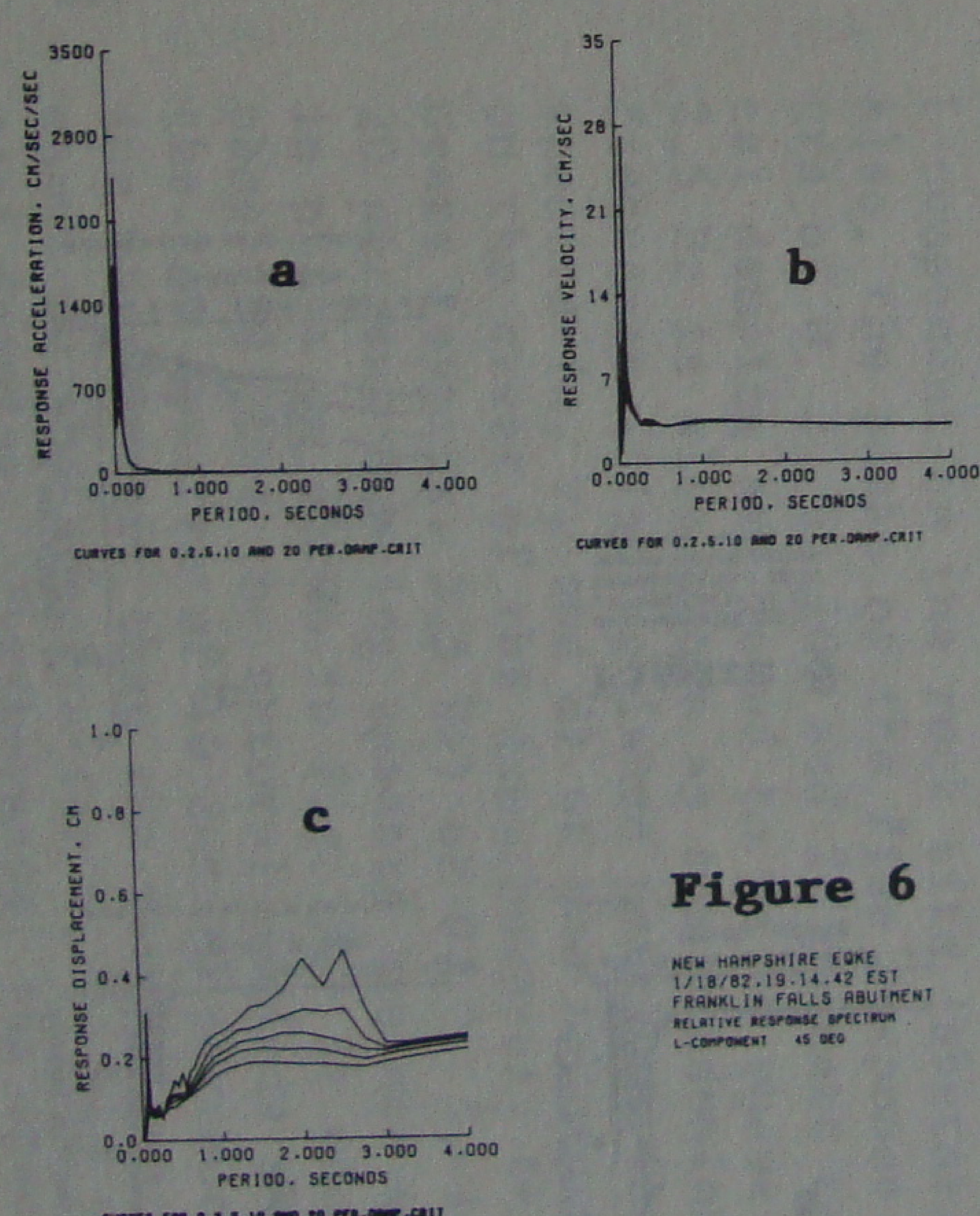

Figure 6

NEW HAMPSHIRE EQKE
1/18/82.19.14.42 EST
FRANKLIN FALLS ABUTMENT
RELATIVE RESPONSE SPECTRUM
L-COMPONENT 45 DEG

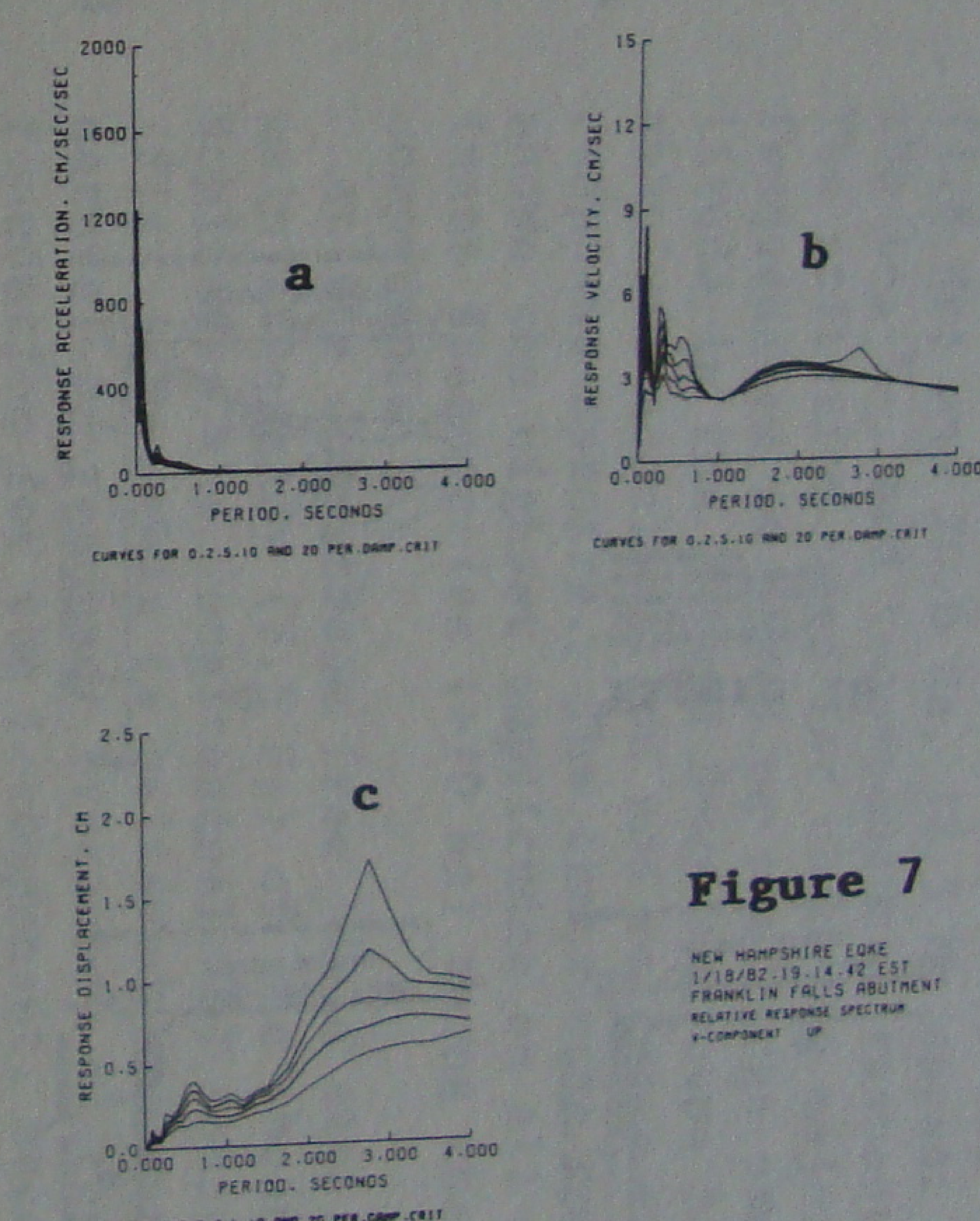

Figure 7

NEW HAMPSHIRE EQKE
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FRANKLIN FALLS ABUTMENT
RELATIVE RESPONSE SPECTRUM
V-COMPONENT UP

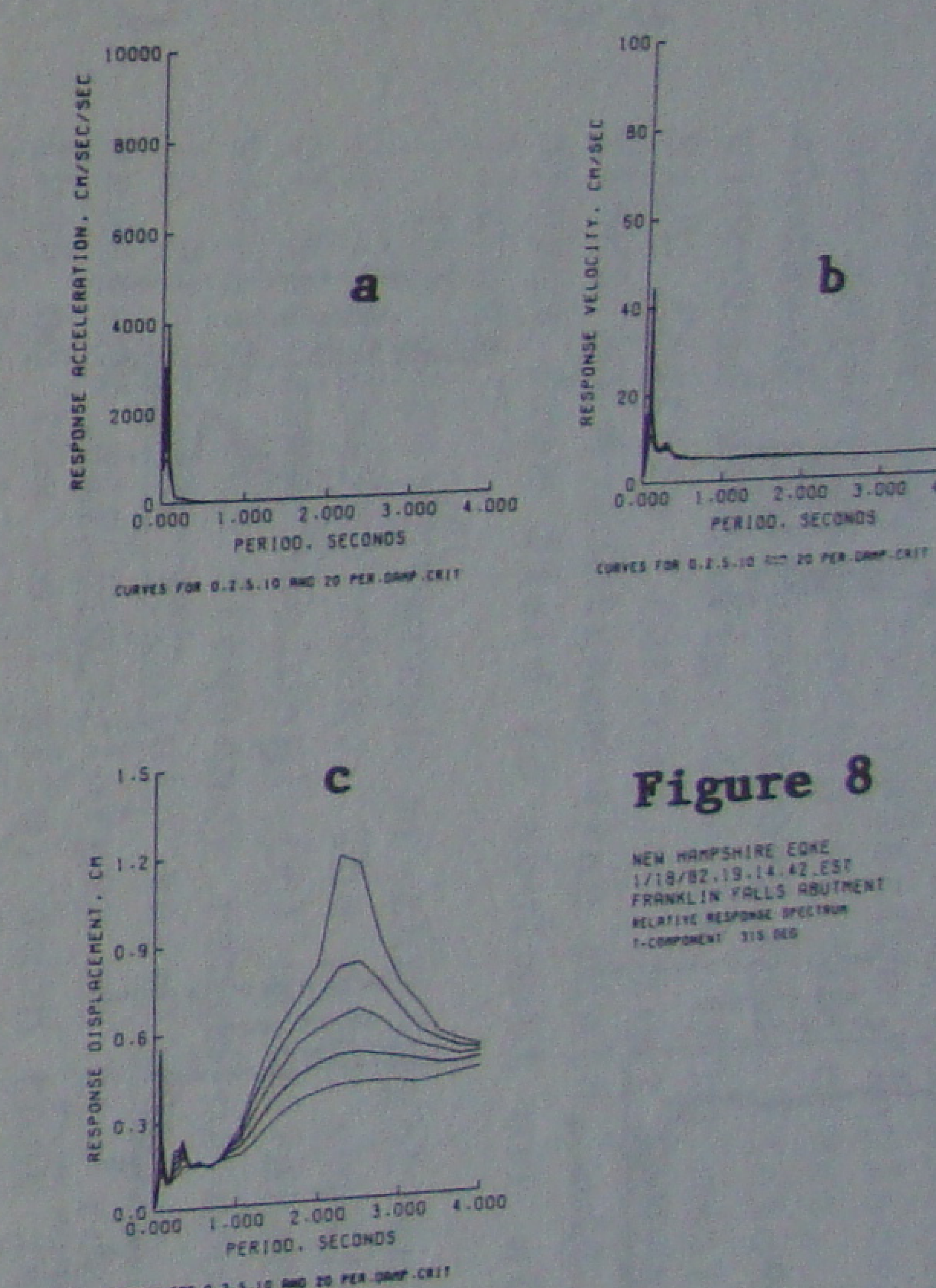

Figure 8

NEW HAMPSHIRE EQKE
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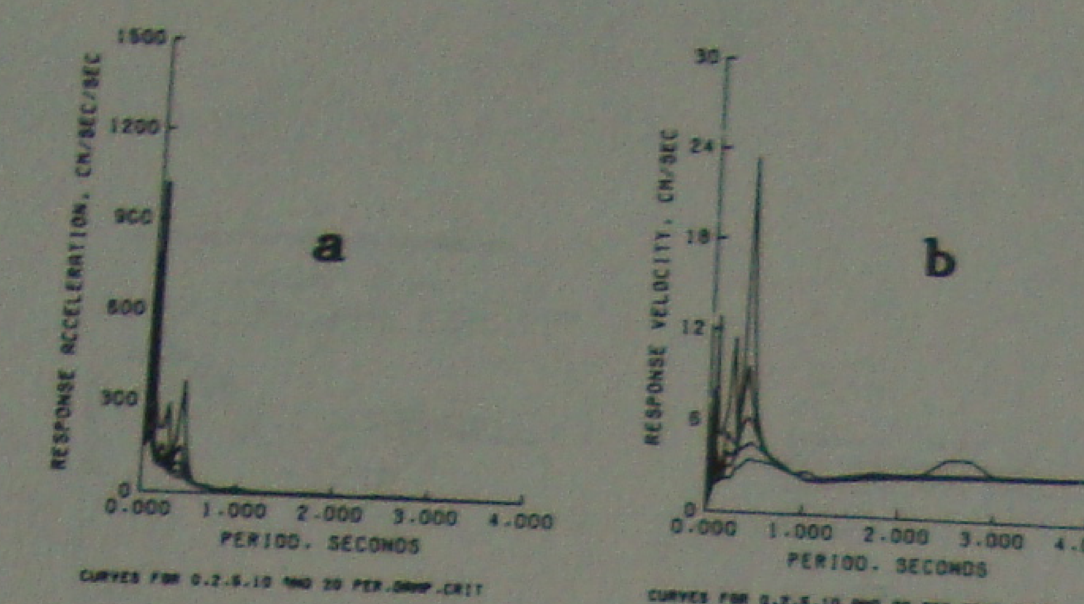
FRANKLIN FALLS ABUTMENT
RELATIVE RESPONSE SPECTRUM
T-COMPONENT 315 DEG



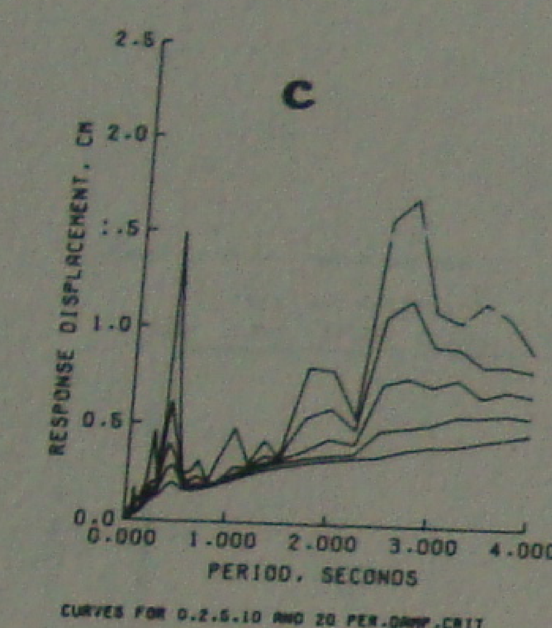
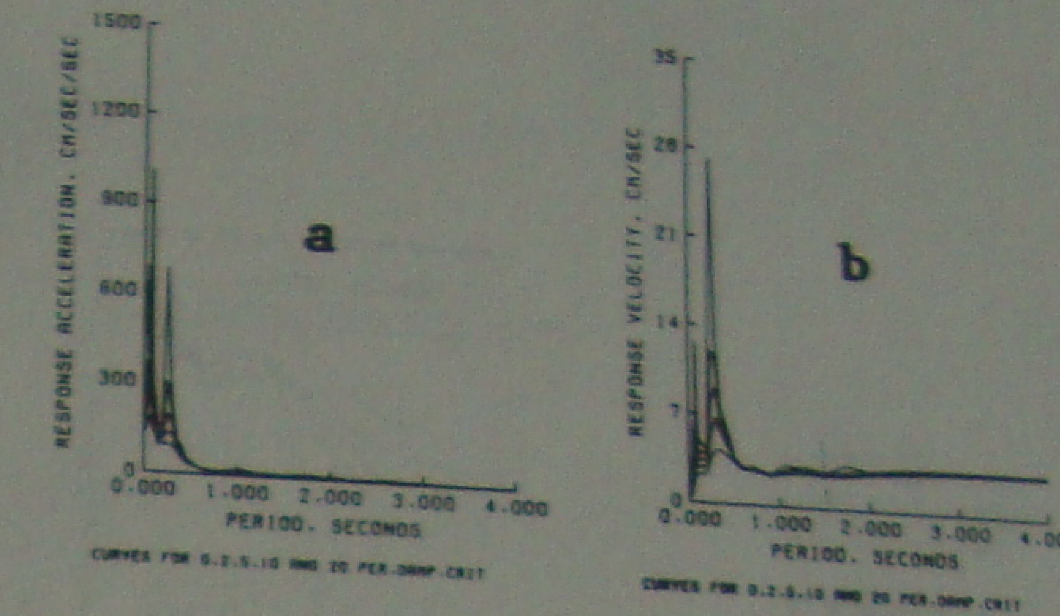


Figure 9

NEW HAMPSHIRE EQKE
1/18/82.19.14.42 EST
FRANKLIN FALLS CREST
RELATIVE RESPONSE SPECTRUM
L-COMPONENT 45 DEG



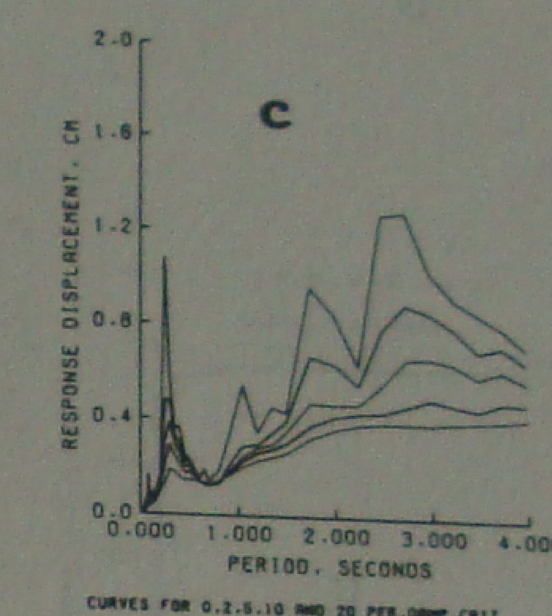


Figure 10

NEW HAMPSHIRE EQKE
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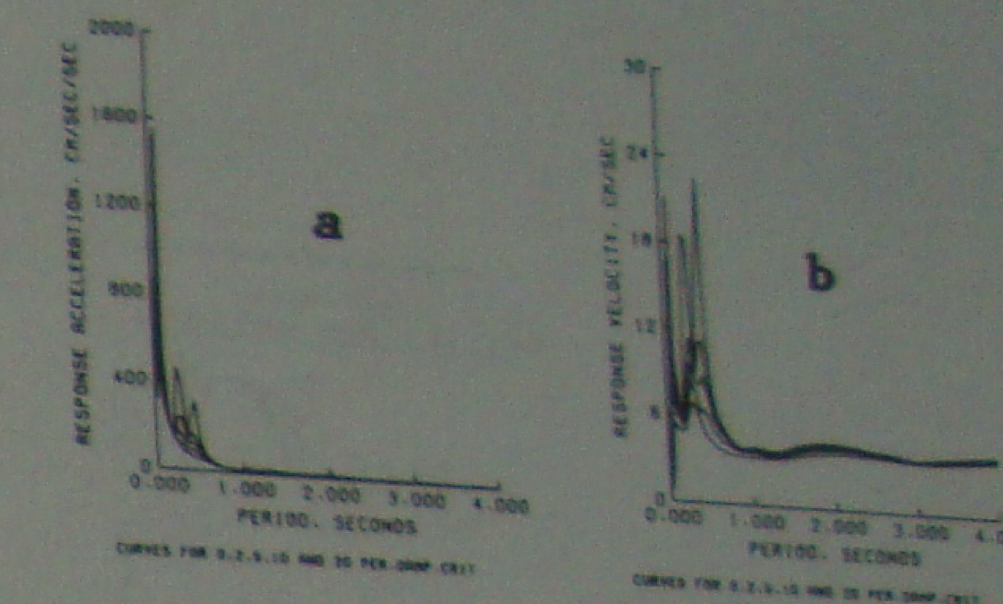
FRANKLIN FALLS CREST
RELATIVE RESPONSE SPECTRUM
V-COMPONENT UP



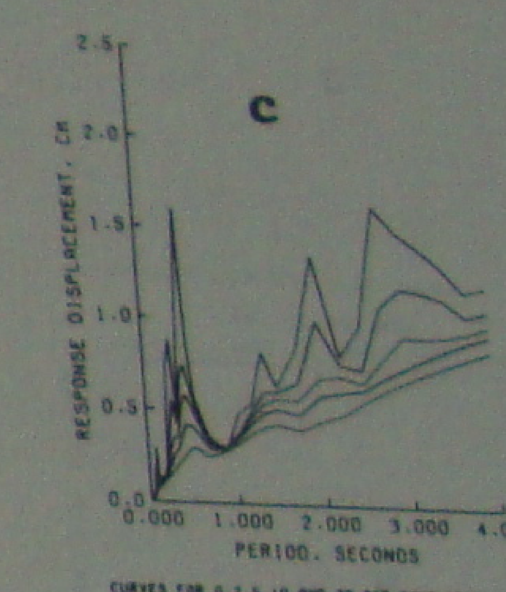


Figure 11

NEW HAMPSHIRE EQKE
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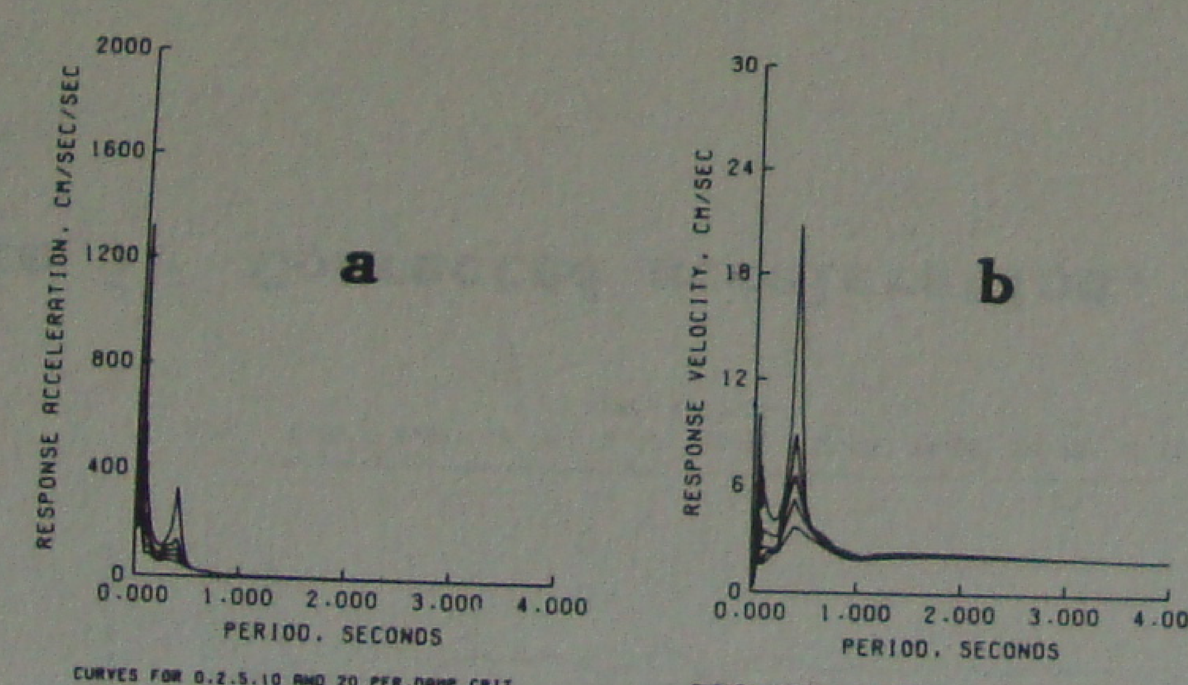
FRANKLIN FALLS CREST
RELATIVE RESPONSE SPECTRUM
T-COMPONENT 315 DEG



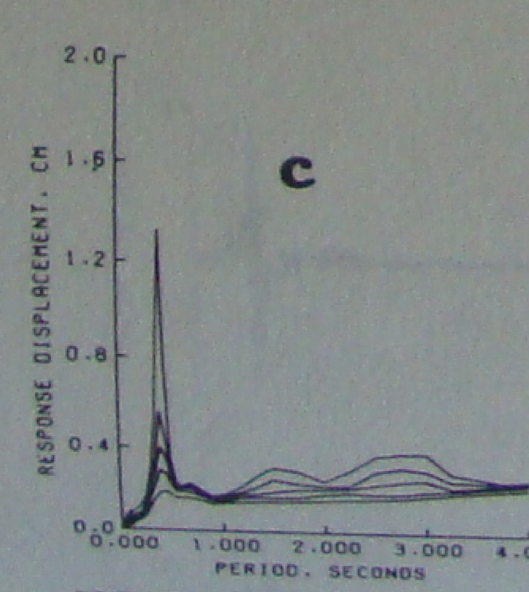


Figure 12

NEW HAMPSHIRE EQKE
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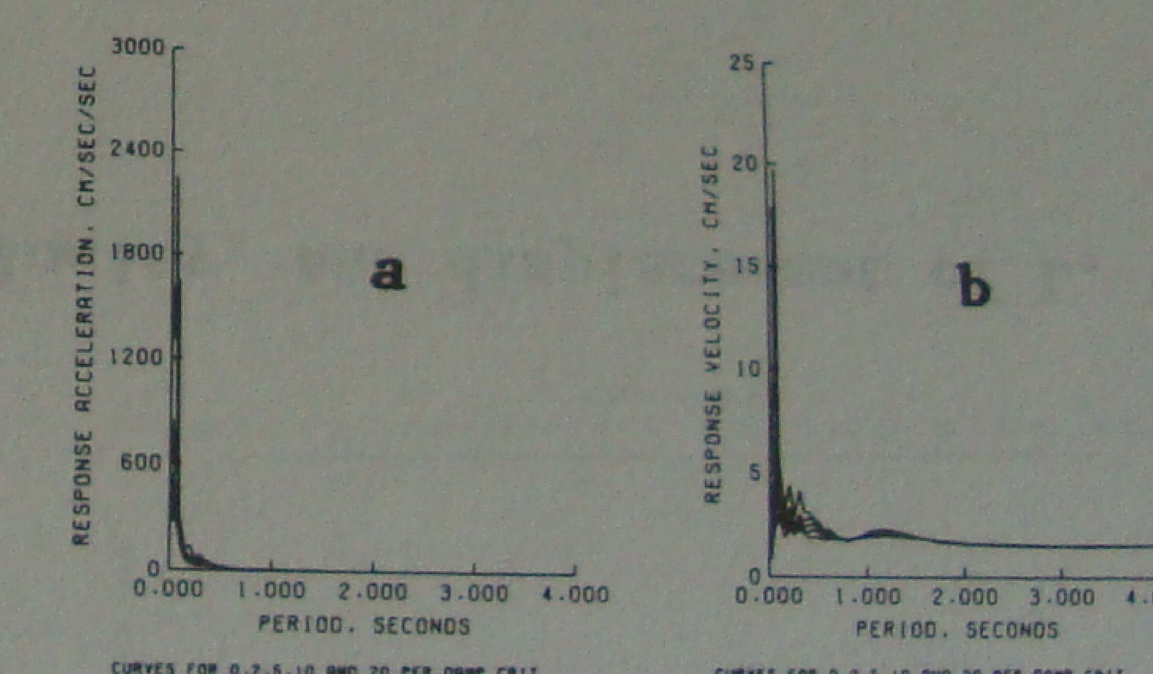
FRANKLIN FALLS DOWNSTREAM
RELATIVE RESPONSE SPECTRUM
L-COMPONENT 225 DEG



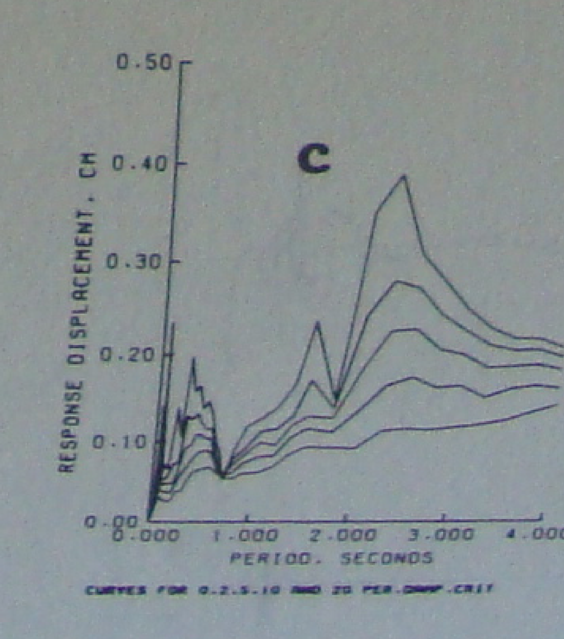


Figure 13

NEW HAMPSHIRE EQKE
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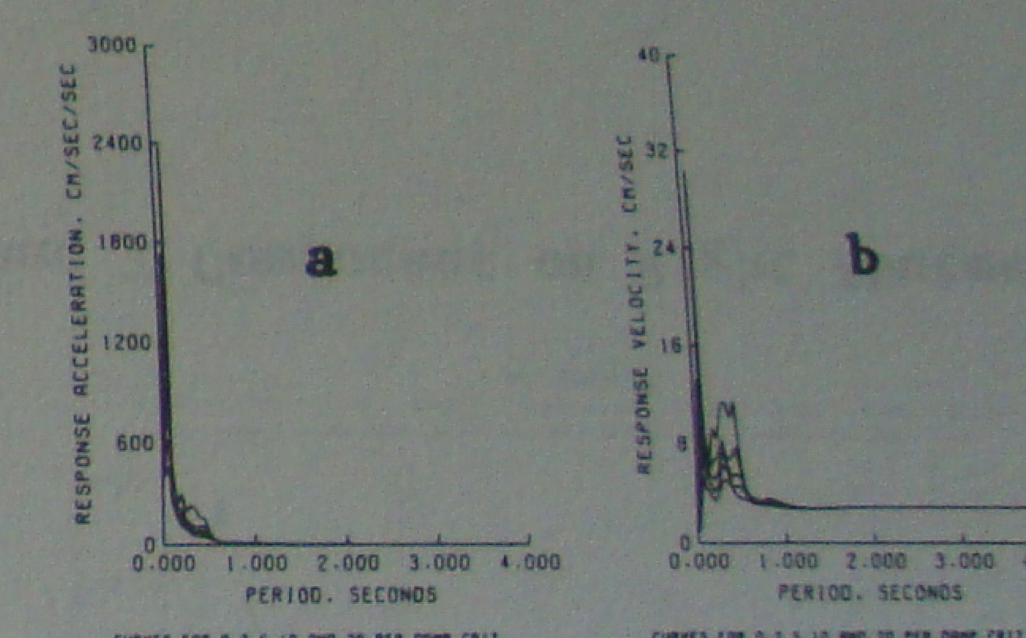
FRANKLIN FALLS DOWNSTREAM
RELATIVE RESPONSE SPECTRUM
V-COMPONENT UP



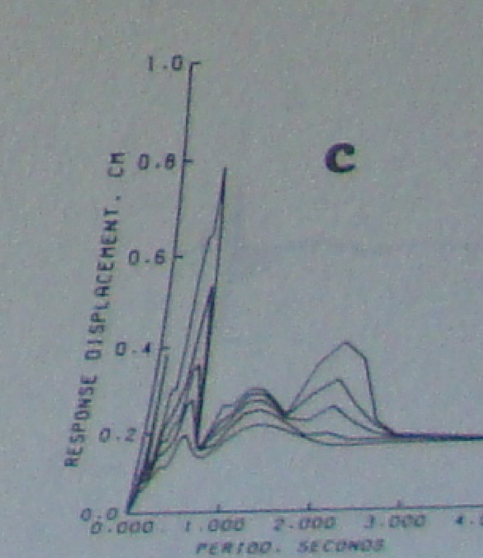


Figure 14

NEW HAMPSHIRE EQKE
1/18/82.19.14.42 EST
FRANKLIN FALLS DOWNSTREAM
RELATIVE RESPONSE SPECTRUM
T-COMPONENT 135 DEG

the relative displacement response spectrum (Fig. 6c - 14c). When the resonant period appears in the shear waves, the displacement amplitudes of compression and other higher mode shear waves are reduced rapidly, as demonstrated in the L- and T-components of the downstream free field station or 2.5 Hz on L-component (Fig. 12c) and 2.2 Hz on T-component (Fig. 14c), respectively. However, the response surface waves of the large periods in the displacement spectrum contain more energy than the shear and compression waves. The compression waves are almost nonexistent in the displacement spectrum.

5.4 Damping ratios and undamped periods of the response spectra

In the engineering design, the response spectral curves for 0, 2, 5, 10, and 20 percent critical damping are usually computed and plotted for absolute acceleration, relative velocity, and relative displacement (Fig. 6 - 14), respectively, or in the form of tripartite graphs of pseudovelocity versus undamped natural period, in addition to a comparison of the relative velocity response spectrum of zero damping with the Fourier amplitude spectrum. Usually, spectra for 2, 5, 10, or 20 percent critical damping that are computed for design purposes are a function of the capability of the structure to dissipate energy without deforming beyond some accepted level. When the damping value increases, the maximum amplitude and irregularity of the response spectrum will decrease. Thus, the numbers of peaks are reduced. However, the distinguished normal mode and higher modes will be persistently preserved, especially the resonant amplitude. Sometimes, the fundamental period increases with the increase in damping. Damping actually functions as a low pass filter, filtering out some or all of the high-frequency responses. In soils, the amount of damping is dependent upon the intensity of ground shaking so that the material damping increases with an increase in ground shaking intensity. At 20 percent damping, which for a structure is usually considered high or overdamped, spectra have very few distinguishable peaks.

6 NATURAL FREQUENCIES (PERIODS) OF THE FOUNDATION SOILS

Table 2 summarizes the characteristics of the response spectra of 5 percent damping for L-, V-, and T-components including peak spectral amplitudes of acceleration, relative velocity, relative displacement, and major frequencies at the crest, right abutment, and downstream free field sites, pictured from Fig. 6 - 14. The amplification factors (ratios) of crest to abutment, crest to downstream free field, and downstream free field to abutment are also listed.

The accelerograph at the right abutment is located on rock. The peak amplitudes of the compressional (P) or dilatational waves in the frequency range of above 10 Hz on the absolute acceleration response spectrum are much larger on the rock site than on the soil site, such that 14.3 Hz, 12.5 Hz, 11.8 Hz; 14.3 Hz, 12.5 Hz, 11.8 Hz; and 20.0 Hz, 12.5 Hz appear on the L-, T-, and V-components, respectively. The criterion used for choosing major peak frequencies is based on the large peak amplitude in the P- and S-wave groups from the absolute acceleration and relative velocity response spectra. The transverse component shows the highest amplitudes on all predominant frequencies (>10 Hz) among the three components.

At the downstream free field site, the main frequency (2.5 Hz) of the shear wave dominates the longitudinal (L) component (Fig. 12). This is believed to be the natural or resonant frequency of the soil foundation for the L-component, because it is not evident on the spectra of the L-component on the rock site (abutment). Also, the peaks of the five response spectra of 0, 2, 5, 10, and 20 percent critical damping are found to be in phase. The 2.5 Hz is also shown as the predominant frequency (distinguished peak) on the spectra (a, v, and d) of the L-component on the crest of the dam; however, the five peaks (approx. 2.5 Hz) of 0, 2, 5, 10, and 20 percent damping spectra are not exactly in phase as shown in Fig. 9. It was also observed that the natural periods shown on the two spectra of 10 and 20 percent damping were larger than the natural periods shown on the spectra for 0, 2, and 5 percent damping. Furthermore, the spectral amplitude at 2.5 Hz of the L-component on the crest is somewhat less than the amplitude of the L-component at the downstream site due to the attenuation of energy. Even though these are the facts, we cannot say that the 2.5 Hz is not the natural frequency of the combined dam-foundation system.

After a detailed examination of the response spectral frequencies on the curves of 10 and 20 percent damping (Fig. 9b) of the L-component on the crest, we found an interference of two other fre-

Table 2

Analyses of Peak Response Spectra (5% Damping) of the Franklin Falls Dam

New Hampshire Earthquake of January 18, 1982

Instrument Component	Peak Frequency Hz	Right Abutment			Downstream (Toe)			Center Crest			Amplification Factor								
											Ctr Crest/ Rt Abutment			Ctr Crest/ Downstream			Downstream/ Rt Abutment		
		a cm/sec^2	v cm/sec	d cm	a cm/sec^2	v cm/sec	d cm	a cm/sec^2	v vm/sec	d cm	a	v	d	a	v	d	a	v	d
	20.0	700.120	4.930	0.041	523.591	3.969	0.030	289.322	1.825	0.017	0.413	0.424	0.415	0.553	0.527	0.567	0.748	0.805	0.732
	18.2	616.117	4.595	0.039	509.827	4.088	0.032	295.691	2.091	0.019	0.480	0.397	0.487	0.580	0.446	0.594	0.827	0.889	0.821
	14.3	1228.955	13.273	0.147	223.574	2.654	0.027	309.033	2.814	0.037	0.251	0.212	0.252	1.382	1.060	1.370	0.182	0.200	0.184
	12.5	1096.469	13.569	0.155	236.147	2.608	0.033	353.712	3.696	0.050	0.322	0.272	0.322	1.498	1.417	1.515	0.215	0.192	0.213
L	11.8	909.465	11.990	0.146	218.600	2.652	0.035	343.421	4.022	0.055	0.377	0.335	0.377	1.571	1.516	1.571	0.240	0.221	0.240
	11.4	694.302	10.574	0.126	171.982	2.458	0.031	306.149	3.945	0.056	0.441	0.373	0.444	1.780	1.605	1.806	0.248	0.232	0.246
	5.0	73.416	3.964	0.073	69.049	2.379	0.069	117.300	3.963	0.118	1.598	0.999	1.616	1.699	1.666	1.710	0.940	0.600	0.945
	4.0	37.863	3.280	0.058	65.877	2.613	0.104	101.553	3.614	0.160	2.682	1.102	2.759	1.541	1.383	1.538	1.739	0.797	1.793
	3.3	40.077	3.453	0.089	94.333	4.158	0.213	72.384	4.060	0.164	1.806	1.176	1.843	0.767	0.976	0.770	2.354	1.204	2.393
	2.5	26.954	3.431	0.106	100.155	6.578	0.403	97.510	6.227	0.393	3.618	1.815	3.707	0.973	0.947	0.975	3.716	1.917	3.802
	20.0	475.210	3.354	0.028	413.681	2.855	0.024	252.146	1.736	0.016	0.530	0.517	0.571	0.609	0.608	0.666	0.870	0.851	0.857
	12.5	364.140	4.087	0.052	474.064	5.931	0.067	385.332	4.440	0.062	1.058	1.058	1.192	0.813	0.749	0.925	1.302	1.451	1.228
	5.0	59.493	2.547	0.060	48.531	2.914	0.049	123.625	3.357	0.125	2.078	1.318	2.083	2.547	1.152	2.551	0.816	1.144	0.817
V	4.0	75.268	3.829	0.118	40.574	2.157	0.063	194.679	8.100	0.307	2.586	2.115	2.601	4.798	3.755	4.873	0.539	0.563	0.534
	3.3	67.317	4.561	0.152	40.375	2.714	0.092	170.444	9.106	0.387	2.532	1.996	2.546	4.221	3.355	4.206	0.559	0.595	0.605
	2.5	46.843	3.419	0.186	27.303	2.480	0.109	63.943	5.473	0.257	1.365	1.600	1.382	2.342	2.207	2.358	0.583	0.725	0.586
	0.57	5.304	3.286	0.409	1.842	1.721	0.128	6.149	2.829	0.475	1.159	0.861	1.161	3.338	1.644	3.711	0.347	0.524	0.313
	0.40	5.350	3.148	0.842	1.459	1.564	0.229	3.557	3.243	0.559	0.665	1.030	0.664	2.438	2.073	2.441	0.273	0.497	0.272
	14.3	1981.993	20.884	0.238	797.950	8.776	0.096	905.729	10.112	0.109	0.457	0.484	0.458	1.135	1.152	1.135	0.403	0.420	0.403
	12.5	1893.661	22.628	0.269	827.043	10.177	0.117	767.950	9.874	0.108	0.406	0.436	0.401	0.928	0.976	0.923	0.437	0.447	0.435
	11.8	1861.172	23.571	0.301	816.506	10.753	0.132	703.095	9.451	0.113	0.378	0.401	0.375	0.861	0.879	0.856	0.439	0.456	0.438
	11.1	1436.281	21.786	0.280	621.746	9.832	0.122	602.112	8.792	0.117	0.419	0.404	0.418	0.968	0.894	0.959	0.433	0.451	0.436
T	5.0	104.690	7.207	0.104	179.701	5.086	0.181	142.594	5.392	0.142	1.362	0.748	1.365	0.793	1.060	0.784	1.716	0.706	1.740
	4.0	101.779	8.289	0.159	116.633	5.634	0.182	177.926	6.917	0.280	1.748	0.834	1.761	1.525	1.228	1.538	1.146	0.679	1.145
	3.3	75.890	8.778	0.169	97.103	7.405	0.220	149.457	9.595	0.339	1.969	1.093	2.006	1.539	1.296	1.541	1.279	0.844	1.302
	2.5	45.714	5.928	0.176	86.298	5.300	0.347	129.676	8.100	0.523	2.837	1.366	2.971	1.503	1.528	1.507	1.888	0.894	1.971
	2.2	31.376	5.709	0.149	71.207	5.544	0.363	113.709	8.737	0.580	3.624	1.530	3.893	1.597	1.576	1.598	2.269	0.971	2.436
	0.4	4.185	5.050	0.655	1.553	2.903	0.195	4.740	4.086	0.670	1.133	0.809	1.023	3.052	1.407	3.436	0.371	0.575	0.298

quencies of 2.65 Hz and 2.29 Hz. Because both frequencies are so close to 2.5 Hz, when the damping ratio increases in the response spectrum, the shape of the spectrum becomes more flat. Thus, the fundamental period will be slightly shifted to the right. This is due to the natural characteristics of the undamped single degree of freedom system, which is a viscously damped, simple oscillator subjected to the base motion. The largest spectral amplitude ratios of downstream free field to abutment for the L-, V-, and T-components are at 2.5, 3.3, and 2.2 Hz, respectively. Therefore, these are believed to be the natural frequencies of the foundation for these components.

7 NATURAL FREQUENCY (PERIOD) OF THE DAM

7.1 Analysis of response spectra

By comparing the shear wave spectral amplitude ratios or amplification factors from 2.0 to 7.0 Hz of the acceleration, velocity, and displacement response spectra of the L-, V-, and T-components on the crest to those at the abutment and downstream sites, one finds the largest spectral amplitude ratios on the L-, V-, and T-components at 2.5, 4.0, and 2.2 Hz, respectively. Although the 2.5-Hz peak of the longitudinal component has the largest spectral amplitude on the crest, it is still less than the spectral amplitude at 2.5 Hz of the L-component at the foundation. This was possibly caused by a higher rate of energy dissipation for the longitudinal component of the dam's vibration or by a higher material damping which was caused by the stronger ground-shaking intensity for the L-component. The highest peak spectral amplitude at 2.5 Hz on the L-component has to be the fundamental mode (natural period) for the L-component of the dam. The second highest acceleration spectral amplitude ratio, 2.7, of crest to abutment on the L-component is at 4.0 Hz. This indicates that 4.0 Hz is the second natural frequency for the L-component of the dam since it is much higher than the amplitude ratio 1.7 of downstream to abutment. The natural frequency (first mode) for the V-component is 4.0 Hz; this frequency shows the largest spectral amplitude ratio on both crest to abutment and crest to downstream. For the T-component, 2.2 Hz is the natural frequency for both foundation and dam; the resonant spectral amplitude on the crest is much higher than on the foundation (downstream site). In conclusion, the natural frequencies of the Franklin Falls Dam are 2.5 Hz, 4.0 Hz, and 2.2 Hz for L-, V-, and T-components, respectively.

7.2 Shear beam method

Gazetas (1981a, b, 1982) formulated the natural periods for longitudinal and vertical deformations of an earth dam based on the generalized homogeneous equations:

$$T_{\eta,h} = \frac{2.57}{\eta}\,\frac{H}{\bar{V}}\ ;\ \eta = 1, 2, 3, \ldots \quad (1)$$

where

$T_{\eta,h}$ = the natural period for longitudinal deformations

H = the height of the dam

$\bar{V}$ = the average S-wave velocity of the dam;

and

$$T_{\eta,v} = \sqrt{\lambda}\,T_{\eta,h}\ ;\ \eta = 1, 2, 3, \ldots \quad (2)$$

where

$T_{\eta,v}$ = the natural period for vertical deformations

$$\lambda = \frac{1-\nu}{2} \quad (3)$$

ν = Poisson's ratio

The average height of the Franklin Falls Dam is about 40 m, and the observed natural period of the L-component is 0.4 sec. Then, substituting these two parameters into Equation 1, we obtain 257 m/sec as an estimate of the average shear wave velocity in the dam.

If the average Poisson's ratio of the Franklin Falls Dam is assumed to be 0.3, then in Equation 3, $\lambda = 0.35$ and $\sqrt{\lambda} = 0.5916$; therefore, substituting this in Equation 2, we have

$T_{1,v} = 0.5916 \times T_{1,h}$ (n = 1, normal mode of V-component)

Since $T_{1,h} = 0.4$ sec for the L-component, then

$$T_{1,v} = 0.5916 \times 0.4$$
$$= 0.24 \text{ sec } (4.1 \text{ hz})$$

Hence, the calculated natural period, 0.24 sec (4.1 Hz) of V-component based on the assumed ν = 0.3 is generally in agreement with the observed natural period, 0.25 sec (4 Hz) of V-component of the Franklin Falls Dam.

Therefore, based on the measured natural period of the horizontal component and average height of the dam, the average shear velocity and Poisson's ratio of the dam can be estimated. A useful check on this analysis could be made by comparing the estimated values with the results of actual field measurement, so far not available.

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